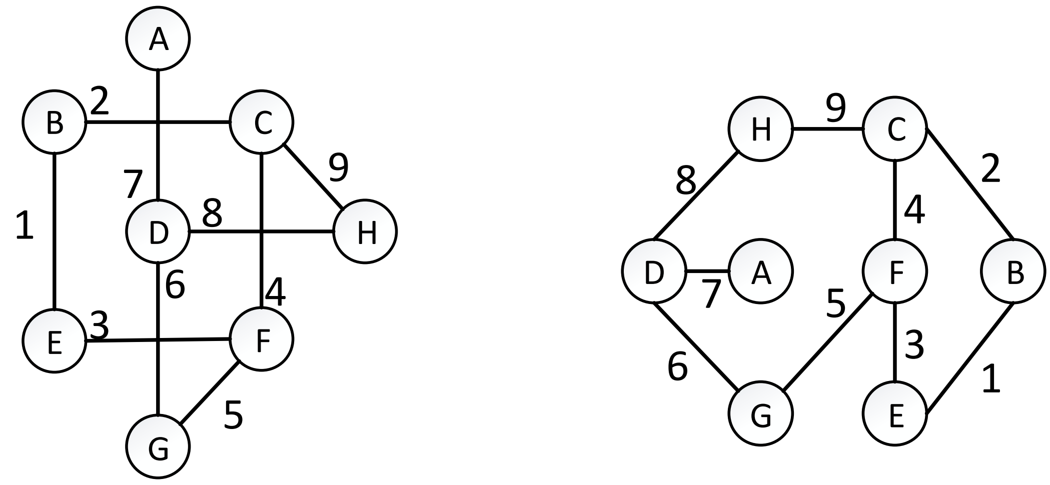
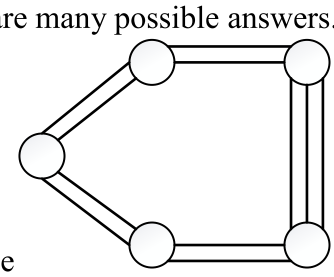
PART A – GRAPH THEORY – 20 MARKS

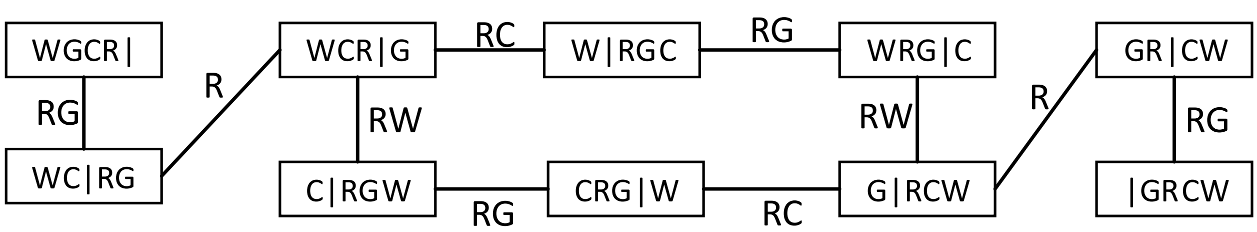
1. Equivalent Graphs (4 marks)



2. Graph Degrees (6 marks)

a) A graph with 5 vertices of degrees 5, 5, 4, 4, 3 This is not possible because the total degree of this graph would be 21 which is odd. By the Handshake theorem graphs must have even degrees,	b) A graph with 5 vertices of degrees 5, 5, 4, 4, 4 There are many possible answers. For example 

3. Problem Solving with Graphs (10 marks)



PART B – SEQUENCES, RECURRENCE RELATIONS – 10 MARKS

1. Terms of a Sequence (6 marks)

$a_2 = a_1 + \frac{1}{2(3)}$ = ½ + 1/(2×3) = ½ × (1+1/3) = ½ × 4/3 =2/3

$a_3 = a_2 + \frac{1}{3(4)}$ = 2/3 + 1/(3×4) = 1/3 × (2+1/4) = 1/3 × 9/4 =3/4

$a_4 = a_3 + \frac{1}{4(5)}$ = 3/4 + 1/(4×5) = 1/4 × (3+1/5) = 1/4 × 16/5 =4/5

2. Iteration (4 marks)

$a_n = n / (n+1)$

PART C – INDUCTION – 20 MARKS

1. <u>Mathematical (weak) Induction (10 marks)</u>

Prove by induction that for all positive integers n, $\sum_{i=2}^{n} i(i-1) = \frac{n(n-1)(n+1)}{3}$

Let P(n) be the property: $\sum_{i=2}^{n} i(i-1) = \frac{n(n-1)(n+1)}{3}$

We must prove $\forall n \in \mathbb{N}^+$ P(n)

Proof by induction

<u>Base case</u>:

Let n=1, then $\sum_{i=2}^{n} i(i-1) = \sum_{i=2}^{0} i(i-1) = 0$

Also $\frac{n(n-1)(n+1)}{3} = \frac{1(0)(2)}{3} = 0$

So P(0) is true

<u>Inductive step</u>:

Assume that P(k) is true for some k≥1

i.e. $\sum_{i=2}^{k} i(i-1) = \frac{k(k-1)(k+1)}{3}$ (Inductive Hypothesis)

We will show that P(k+1) is also true

i.e. we will show that $\sum_{i=2}^{k+1} i(i-1) = \frac{(k+1)k(k+2)}{3}$

$\sum_{i=2}^{k+1} i(i-1) = (k+1)(k) + \sum_{i=2}^{k} i(i-1)$ Definition of sum

$= k(k+1) + \frac{k(k-1)(k+1)}{3}$ by Inductive Hypothesis

$= 1/3\ [3k(k+1) + k(k-1)(k+1)]$ algebra

$= k(k+1)/3\ [3+(k-1)]$ algebra

$= k(k+1)(k+2)/3$ algebra

QED

2. <u>Types of induction (10 marks)</u>

Proof description	**Needs** strong induction? (Y/N)	Explanation of your answer
Proof of the correctness of the solution for the sequence a_n recursively defined as: $a_1=2$, $a_2=3$, $a_n=a_{n-2}+2$ for n>2	Y	The recurrence relation defines a_n in terms of a_{n-2}, Therefore in order to prove that the solution is correct for a_n, one must assume that the solution is correct for a_{n-2}. Therefore the inductive hypothesis cannot be simply that the solution is correct for a_{n-1}.
Proof of the correctness of the solution for the sequence b_n recursively defined as: $b_1=2$, $b_2=5$, $b_n=b_{n-1}+3$ for n>2	N	Even though this sequence has a recurrence relation that starts at n=3, the recurrence relation is also true for n=2, Therefore this sequence is a simple arithmetic sequence starting at n=1 with a recurrence relation where each term is defined as a function of the previous term. The correctness of the solution $a_n= 2+3(n-1)$ can be proved using mathematical induction.
Proof that every positive integer has a unique binary representation starting with a leading 1.	Y	The proof of the existence part of this theorem is a proof by induction which proves the property for an integer n by assuming that it is true for $\lfloor n/2 \rfloor$. Therefore the inductive hypothesis must be that the property is true for all number less than n.